Testing the Kibble-Zurek Scenario with Annular Josephson Tunnel Junctions

E. Kavoussanaki[a*], R. Monaco[b†] and R. J. Rivers[a‡]
a)Theoretical Physics, Blackett Laboratory, Imperial College, Prince Consort Road, London, SW7 2BZ, U.K.
b) Istituto di Cibernetica del C.N.R., I-80072, Arco Felice (Na), Italy and INFM-Dipartimento di Fisica, Universita' di Salerno, I-84081 Baronissi (Sa), Italy

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In parallel with Kibble's description of the onset of phase transitions in the early universe, Zurek has provided a simple picture for the onset of phase transitions in condensed matter systems, supported by agreement with experiments in ^3He and superconductors. In this letter we show how experiments with annular Josephson tunnel Junctions can, and do, provide further support for this scenario.

As the early universe cooled it underwent a series of spontaneous phase transitions, whose potential inhomogeneities (monopoles, cosmic strings, domain walls) have observable consequences, for structure formation in particular. These defects appear because the correlation length ξ of the field (or fields) whose expectation value is the order parameter is necessarily *finite* for a transition that is implemented in a finite time

Using *nothing more* than simple causal arguments Kibble [1,2] made estimates of this early field ordering, and the density of topological defects produced at GUT transitions at $10^{-35}s$. Unfortunately, because the effects of their evolution are not visible until the decoupling of the radiation and matter $10^6 yrs$ later, it is impossible to provide unambiguous checks of these predictions. However, causality is such a fundamental notion that Zurek suggested [3,4] that identical causal arguments, with similar predictions, were applicable to condensed matter systems for which direct experiments on defects could be performed. The hope is that successful tests of these predictions could lead to a better understanding of phase transitions in quantum fields.

Whether for the early universe or condensed matter, consider a quench of the system in which its temperature $T(t)$ is reduced as time passes. In the vicinity of the critical temperature T_c we assume that the temperature T decreases linearly with the time t at a rate $dT/dt = -T_c/\tau_Q$, τ_Q being the quenching time.

Suppose that the 'equilibrium' correlation length $\xi_{eq}(t) = \xi_{eq}(T(t))$ of the order-parameter field, and its relaxation time $\tau(t)$, diverge at $t = 0$ (when $T = T_c$) as

$$\xi_{eq}(t) = \xi_0 \left|\frac{t}{\tau_Q}\right|^{-\nu}, \;\tau(t) = \tau_0 \left|\frac{t}{\tau_Q}\right|^{-\gamma}. \tag{1}$$

The fundamental length and time scales ξ_0 and τ_0 of a system are determined from its microscopic dynamics. One definition of $\tau(t)$ is that $\bar{c}(t) = \xi_{eq}(t)/\tau(t)$ denotes the maximum speed, at time t, at which the order parameter can change. In quantum field theory $\bar{c}(t) = c_0$, the speed of light in vacuo.

Although $\xi_{eq}(t)$ diverges at $t = 0$ this is not the case for the true non-equilibrium correlation length $\xi(t)$. Kibble and Zurek made two assumptions. Firstly, the correlation length $\bar{\xi}$ of the fields that characterizes the onset of order is the equilibrium correlation length $\bar{\xi} = \xi_{eq}(\bar{t})$ at some time $\bar{t}$ constrained by causality. Secondly, at this time, defects appear with separation $\xi_{def} = O(\bar{\xi})$.

To determine $\bar{t}$ we rephrase the original Kibble-Zurek argument in a way appropriate to Josephson tunnelling Junctions (JTJs), so as only to discuss times $t > 0$. We begin by noting that, in the adiabatic regime away from the transition, static defects can be thought of as kinks, balls, lines[1] or sheets of 'false' vacuum or disordered ground state, of thickness $O(\xi_{eq}(t))$. Thus $\dot{\xi}_{eq}(t) = d\xi_{eq}(t)/dt < 0$ measures the rate at which these defects contract, i.e., the speed of interfaces between ordered and disordered ground states. Since $\dot{\xi}_{eq}(t)$ decreases with time $t > 0$, the *earliest* possible time t at which defects could possibly appear is determined by $|\dot{\xi}_{eq}(t)| = \bar{c}(t)$, given our definition of $\bar{c}(t)$. Although this gives a *lower* bound for $\bar{t}$, as an order of magnitude estimate we identify this time t with $\bar{t}$, whence

$$\tau_0 \ll \bar{t} = (\tau_Q^{\gamma}\tau_0)^{1/(\gamma+1)} \ll \tau_Q. \tag{2}$$

The corresponding *smallest* [4] correlation length is

$$\bar{\xi} = \xi_{eq}(\bar{t}) = \xi_0 \left(\frac{\tau_Q}{\tau_0}\right)^{\nu/(\gamma+1)} \gg \xi_0. \tag{3}$$

*E-mail: `e.kavoussanaki@ic.ac.uk`
†E-mail: `roberto@sa.infn.it`
‡E-mail: `r.rivers@ic.ac.uk`

[1] For ^3He vortices can be more complicated, but our general argument is unaffected.

Because of the qualitative nature of the arguments, factors close to unity are omitted[2]. Further, at the same level of approximation, we shall use mean field critical indices throughout. Measurements [5,6] of *total* vortex density in transitions of ^3He-B support the result (3), when taken together with $\xi_{def} = O(\bar{\xi})$.

As an independent test of the assumptions Zurek suggested using (3) to measure *topological* defect density (in which defects and anti-defects carry opposite weight). This is most easily done in 'one-dimensional' annular geometries, for which experiments were originally proposed [4] with ^4He which, however, has $\bar{\xi}$ so small that the creation of an effectively one-dimensional system is extremely difficult. A recent experiment [7]with annular arrays of high-T_c superconducting islands coupled by grain boundary Josephson junctions confirms part of the picture, but suffers in enforcing a predetermined domain structure. We also wish to check prediction (3) by using annular JTJs. As we shall see, for such JTJs $\bar{\xi}$ is *macroscopically* large, permitting them to be effective one-dimensional systems.

An annular JTJ consists of two superimposed annuli of ordinary superconductors of thickness d_s, separated by a layer of oxide of thickness d_{ox}, whose relative dielectric constant is ϵ_r. Its order-parameter is the relative phase angle $\phi = \theta_1 - \theta_2$ of the complex order parameters $\Psi_1 = \rho_1 \exp(i\theta_1)$ and $\Psi_2 = \rho_2 \exp(i\theta_2)$ of the two superconductors (labelled 1 and 2). After the transition has been implemented, in the adiabatic regime at temperature T, ϕ satisfies the dissipative, one-dimensional sine-Gordon (SG) equation

$$\frac{\partial^2\phi}{\partial x^2} - \frac{1}{\bar{c}^2(T)}\frac{\partial^2\phi}{\partial t^2} - \frac{b}{\bar{c}^2(T)}\frac{\partial\phi}{\partial t} = \frac{1}{\lambda_J^2(T)}\sin\phi, \quad (4)$$

with periodic boundary conditions [8]; x measures the distance along the annulus, its width $w \ll \lambda_J(T)$ being ignored and b is a characteristic frequency that accounts for the viscous drag. The velocity $\bar{c} = \bar{c}(T)$, which depends on the nature of the junction, is the Swihart [9] velocity, the speed of light in a superconducting-insulating-superconducting transmission line. In the Josephson context, it determines the maximum speed at which the order parameter ϕ can change.

The topological defects of the JTJ, the solitons of the sine-Gordon theory, are termed *fluxons*. Their static equilibrium thickness is the Josephson coherence length $\lambda_J(T)$, which plays the role of $\xi_{eq}(T)$ earlier.

Let us attempt to repeat the Kibble-Zurek analysis directly on quenching a JTJ with quench time τ_Q. For simplicity we begin with a symmetric JTJ, in which the electrodes are made of identical materials with common critical temperatures T_c. At time t after the transition $\lambda_J(t) = \lambda_J(T(t))$ is given by

$$\lambda_J(t) = \sqrt{\hbar/2e\mu_0 d_e(t) J_c(t)}. \quad (5)$$

in which J_c is the critical Josephson current density. In (5) $d_e(t)$ is the magnetic thickness. Specifically, if $\lambda_L(t)$ is the London penetration depth of the two (identical) superconducting sheets, then

$$d_e(t) = d_{ox} + 2\lambda_L(t)\tanh\frac{d_s}{2\lambda_L(t)},$$

where
$\lambda_L(t) = \lambda_L(0)/\sqrt{1-(T(t)/T_c)^4} \simeq \lambda_L(0)/2\sqrt{t/\tau_Q}$. Neglecting the barrier thickness $d_{ox} \ll d_s$, λ_L gives $d_e = d_s$ close to T_c, i.e., the magnetic thickness equals the film thickness and can be set constant in (5).

All the t-dependence of λ_J resides in J_c which, for the symmetric JTJ has the form [10]

$$J_c(t) = \frac{\pi}{2}\frac{\Delta(t)}{e\rho_N}\tanh\frac{\Delta(t)}{2k_BT(t)}. \quad (6)$$

In (6) $\Delta(t)$ is the superconducting gap energy and varies steeply near T_c as

$$\Delta(t) \simeq 1.8\,\Delta(0)\left(1-\frac{T(t)}{T_c}\right)^{1/2} = 1.8\,\Delta(0)\sqrt{\frac{t}{\tau_Q}},$$

and ρ_N is JTJ normal resistance per unit area. Introducing the dimensionless quantity $\alpha = 1.6\Delta(0)/k_BT_C$ whose typical value[3] is between 3 and 5, enables us to write $J_c(t)$ as

$$J_c(t) \simeq \alpha J_c(0)\left(1-\frac{T(t)}{T_c}\right) = \alpha J_c(0)\frac{t}{\tau_Q}. \quad (7)$$

Thus, in the vicinity of the transition,

$$\lambda_J(t) = \xi_0\left(\frac{\tau_Q}{t}\right)^{1/2}, \quad (8)$$

corresponding to $\nu = 1/2$ in (1), where

$$\xi_0 = \sqrt{\frac{\hbar}{2e\mu_0 d_s \alpha J_c(0)}}. \quad (9)$$

On the other hand, for a finite electrode thickness tunnel junction, the Swihart velocity takes the form [10]

$$\bar{c}(t) = c_0\sqrt{d_{ox}/\epsilon_r d_i(t)},$$

where

[2]These results, without any additional factors, were originally obtained by Zurek on considering the time $-\bar{t}$ at which the field freezes in.

[3]$\Delta(0)$ and $J_c(0)$ denote the respective values at $T=0$.

$$d_i(t) = d_{ox} + 2\lambda_L(t)\coth\frac{d_s}{2\lambda_L(t)} \simeq \frac{\lambda_L^2(0)}{d_s}\left(\frac{\tau_Q}{t}\right),$$

near the transition. Thus $\overline{c}(t)$ shows critical slowing down at the transition, as

$$\bar{c}(t) = \bar{c}_0\left(\frac{t}{\tau_Q}\right)^{1/2},$$

where $\bar{c}_0 = c_0\sqrt{d_s d_{ox}/\epsilon_r\lambda_L^2(0)}$. These indices ($\nu = 1/2, \gamma = 1$) are typical of condensed matter systems. The causal constraint gives $\bar{t} = \sqrt{\tau_0\tau_Q}$, with $\tau_0 = \xi_0/\bar{c}_0$. Inserting reasonable values [10] of $\xi_0 = 10\,\mu m$ and $\bar{c}_0 = 10^7\,m/s$, gives $\tau_0 = 1\,ps$, and assuming $\tau_Q = 1\,s$, we find $\bar{t} \simeq 1\mu s$. The causal Josephson penetration length is then

$$\overline{\lambda}_J = \lambda_J(\bar{t}) = \xi_0\left(\frac{\tau_Q}{\bar{t}}\right)^{\frac{1}{2}} = \xi_0\left(\frac{\tau_Q}{\tau_0}\right)^{\frac{1}{4}} = 10\,mm. \quad (10)$$

This $\overline{\lambda}_J$, which should characterise fluxon separation at a quench for a symmetric JTJ is far too large. Fortunately, the manufacture of JTJs typically yields *non-symmetric* devices with more acceptable properties. Suppose the two superconductors, 1 and 2, now have different critical temperatures $T_{c2} > T_{c1}$. Fluxons only appear at temperatures $T < T_{c1}$, from which we measure our time t. At this time

$$\Delta_2(T_{c1}) \simeq 1.8\,\Delta_2(0)\left(1 - \frac{T_{c1}}{T_{c2}}\right)^{1/2},$$

and $\Delta_1(t) \simeq 1.8\,\Delta_1(0)\sqrt{t/\tau_Q}$. The critical Josephson current density $J_c'(t)$ for a non-symmetric JTJ, being proportional to $\Delta_1(t)\Delta_2(t)$ [10], behaves just after the transition as

$$J_c'(t) \approx \left(1 - \frac{T_{c1}}{T_{c2}}\right)^{1/2}\alpha' J_c'(0)\left(\frac{t}{\tau_Q}\right)^{1/2}, \quad (11)$$

where $J_c'(0) = \pi\Delta_1(0)\Delta_2(0)/[\Delta_1(0) + \Delta_2(0)]e\rho_N$, and $\alpha' = [\Delta_1(0) + \Delta_2(0)]/k_B T_{c,1}$, provided $\Delta_2(T_{c,1}) \ll 2\pi k_B T_{c,1}$. This is the case here.
The crucial difference between (11) and (7) is in the critical index. Near $t = 0$, we now find

$$\lambda_J(t) = \xi_0\left(1 - \frac{T_{c1}}{T_{c2}}\right)^{-1/4}\left(\frac{\tau_Q}{t}\right)^{1/4}, \quad (12)$$

where ξ_0 is as in (8), since $J_c'(0)$ is indistinguishable from $J_c(0)$ and α' is comparable to α. For the critical behavior (12) to be valid, rather than (8) we need $1 - T_{c1}/T_{c2} \gg O(\bar{t}/\tau_Q) = O(10^{-6})$, which is always the case. For a typical value $(1 - T_{c1}/T_{c2}) = 0.02$ the critical time $\bar{t}$ is now determined by ($\gamma = 3/4$)

$$\bar{t} = \tau_0^{4/7}\tau_Q^{3/7}\left(1 - \frac{T_{c1}}{T_{c2}}\right)^{-1/7} \simeq 0.24\mu s,$$

with our parameters. In turn,

$$\lambda_J(\bar{t}) \simeq \xi_0\left(1 - \frac{T_{c1}}{T_{c2}}\right)^{-1/4}\left(\frac{\tau_Q}{\tau_0}\right)^{1/7} \simeq 1.4mm \quad (13)$$

is an order of magnitude smaller than $\bar{\lambda}_J$ of (10).

While new experiments are required, old experiments on JTJs by one of us [11] are compatible with these predictions, although their specific parameters are not optimal. In these experiments non-symmetric annular $Nb/Al - AlOx/Nb$ JTJs ($T_{c,2}/T_{c,1} - 1 \approx 0.02$) with circumference $C = 0.5\,mm$ were quenched with a quench time $\tau_Q = O(1s)$. The intention was, primarily, to produce fluxons for further experiments, and the density at which they were produced was secondary. From the parameters quoted in [11] for sample B, we estimated $\xi_0 \simeq 6.5\,\mu m$, $\bar{c}_0 \simeq 10^7\,m/s$ and $\tau_0 \simeq 0.65ps$. Inserting these specific values in (13) gives $\bar{\lambda}_J \simeq 1mm$ (with experimental uncertainty of up to 50%). Although $C \simeq \bar{\lambda}_J$ we would have expected to see a fluxon a few percent of the time, given that the variance Δn in the number of fluxons is $\Delta\phi/2\pi$. Indeed, in practice (invariably single) defects formed once every 10-20 times.

We have no detailed knowledge of how the cooling takes place, but do not expect temperature inhomogeneities to be important. The critical slowing down of $\bar{c}(t)$ provides a necessary condition for defects to survive inhomogeneity [12] and, with empirically comparable ξ_0, τ_0, and τ_Q, the situation is no better or worse for JTJs than with any other superconducting system undergoing a mechanical quench. Other samples of the same circumference but with different $\bar{\lambda}_J$ have been tested. Although none had $C/\bar{\lambda}_J$ large, it was observed that the likelihood of seeing a fluxon was greater the larger its value, as we would have predicted, although this was not quantified. This suggests that temperature inhomogeneities are not the direct cause of the observed fluxons.

There are theoretical, as well as experimental, uncertainties. The SG equation (4) can only make sense once the individual superconductors have adjusted themselves. Repeating Zurek's analysis of the Gross-Pitiaevsky equation[4] for individual superconductors [4] gives a minimum time at which the sine-Gordon equation is valid of $\bar{t}_S = \sqrt{\overline{\tau}_0\tau_Q}$ where $\overline{\tau}_0$ in (9) is now determined [4] from Gorkov's equation as $\overline{\tau}_0 = \pi\hbar/16k_B T_c \approx 0.15ps$ for $T_c \approx 10\,K$. The resulting $\bar{t}_S \approx 0.4\mu s$ is commensurate with the values of $\bar{t}$ for the typical symmetric and non-symmetric cases, falling between them. Whereas this suggests that the SG equation is valid for symmetric JTJs at time $\bar{t}$, it also suggests that we should evaluate $\lambda_J(t)$ at $\bar{t}_S$, rather than $\bar{t}$ for the non-symmetric case. However, for our typical parameter values the difference between $\lambda_J(\bar{t})$ and

[4] Justified here by the success of Feynman's coupled model equations for Ψ_1 and Ψ_2. For example, see Ref.10.

$$\lambda_J(\bar{t}_S) \simeq \xi_0 \left(1 - \frac{T_{c1}}{T_{c2}}\right)^{-1/4} \left(\frac{\tau_Q}{\bar{\tau}_0}\right)^{1/8} \simeq 1.1mm \quad (14)$$

is so small as to be ignorable, given the crudity of the bounds. For the specific sample B of [11] the decrease is similar, at $\lambda_J(\bar{t}_S) \simeq 0.7mm$, and equally ignorable.

Further, although the prediction (3), together with $\xi_{def} = O(\bar{\xi})$, has been taken, without additional qualification, as the direct basis for the successful experiments [5,6] in ^3He, and experiments in ^4He [13,14] and high-T_c superconductors [15], the causality argument that we have presented here is very simplistic. For superfluids obeying time-dependent Ginzburg-Landau (TDGL) theory (and QFT) we know [16] that, at early times, the length $\bar{\xi}$ is, correctly, the correlation length of the fields when they have frozen in after the transition. However, we also know [17,18] that the separation of defects is determined largely by the separation of the zeroes of the fields which define their cores. The separation of zeroes is a function of the *short-range* behavior of the correlation functions [17,18], rather than the long-range behavior that determines $\bar{\xi}$. A priori, $\bar{\xi}$ does not characterize defect separation.

Nonetheless, several numerical [19,20] and analytic calculations [16,21,22], based on TDGL theory, have confirmed that the critical index of (3) is, indeed, the correct behavior for defect separation. The reason why this is so is essentially a matter of dimensional analysis. The density of zeroes is, approximately, a *ratio* of moments of the power in the field fluctuations, at early times at least. This leads to strong cancellations of the effects of the microscopic interactions of the system in question.

At the same time, the critical time $\bar{t}$ characteristically underestimates the time at which the order parameter achieves its equilibrium magnitude, which is a more sensible time to begin to count defects. However, if these other systems are a guide [16,21,22] the true time $t^*$is $t^* = O(\bar{t})$, since the unstable long wavelength modes that set up large scale ordering have amplitudes that grow exponentially. As a result any new scales only occur logarithmically in $t^*/\bar{t}$. In fact, a limited calculation, with ^4He in mind, suggests [23] that $\xi_{def}(t^*) \approx \bar{\xi}$ when counting topological density on an annulus. Thus, although the causality bounds are not saturated, their consequences (2) and (3) survive qualitatively and justify experimental confirmation.

That causality is now seen as a constraint, but not the microscopic mechanism, helps explain why the most recent ^4He experiment [14] failed to see any vortices. A major reason (nothing to do with causality) is that vortices are most likely to decay much more rapidly [22] than they were originally thought to do. However, because the ^4He quenches take place entirely within the Ginzburg regime, thermal fluctuations make individual defects scale dependent [21,22], and simple dimensional analysis most likely breaks down, in a way that causality would not have suggested. This is not the case with superconductors, for which the Ginzburg regime is very small.

For that and other reasons, we are sufficiently optimistic to be currently examining the feasability of fabricating JTJs with larger values of $C/\bar{\lambda}_J$ with which to perform new experiments.

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